MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2019/2020

PCM0235 - CALCULUS

(Foundation in Information Technology/ Foundation in Life Sciences)

5 MARCH 2020
2.30 p.m. – 4.30 p.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 2 pages excluding the cover page and appendix.
2. Answer **all FIVE** questions. Each question carries equal marks and the distribution of the marks is given.
3. Write all your answers in the Answer Booklet provided.

Question 1 [10 marks]

a. Find the following limits.

i. $\lim_{x\to\infty}\frac{73+x^3-x^4}{-0.5+3x^2+7x^3}$ (2 marks)

ii. $\lim_{x\to 2}\frac{x^2+3x-10}{x-2}$ (2 marks)

iii. $\lim_{x\to 0}\frac{\sqrt{x^2+9x+9}-3}{x}$ (3 marks)

b. Is the following function continuous? Give reason for your answer.

$$f(x)=\begin{cases}x^3-1 & for \quad x<3\\ x^2+14 & for \quad x\geq 3\end{cases}$$ (3 marks)

Question 2 [10 marks]

a. Find the derivative of the following functions.

i. $y=x^4\left(\sin x^3-\cos x^2\right)$ (2 marks)

ii. $4x^2y^7-2x=x^5+4y^3$ (3 marks)

b. Evaluate the following integral:

$$\int_{-1}^{0}\frac{x^2+7x}{(x+2)(x-1)(x-4)}dx$$ (5 marks)

Question 3 [10 marks]

a. Given $f(x)=x^3-3x^2-9x+5$.

i. Identify the critical numbers and classify these critical points as local maximum, local minimum or neither. (3 marks)

ii. Determine the intervals on which the function is concave up and concave down and the x-coordinate of the inflection point(s). (3 marks)

b. You want to construct a box with a square base and you have only 10 m^2 of material to use in the construction of the box. Assuming that all the material is used in the construction process, what are the dimensions the box can have that give maximum volume. (4 marks)

Continued...

Question 4 [10 marks]

a. Given the curves $x = y^2$ and $x = 8 - y^2$.

i. Find the point of intersection of these curves. (2 marks)

ii. Find the area of the region bounded by these curves. (3 marks)

b. A cup-like object is made by rotating the area between $y = 2x^2$ and $y = x + 1$ with $x \geq 0$. Find the volume of the material needed to make the cup? (5 marks)

Question 5 [10 marks]

a. Find the unique solution for the first order differential equation $y' + 3y = e^{-2t}$ where $y(0) = 5$. (6 marks)

b. Find the general solution for the second order differential equation $y'' + y' - 12y = 0$. (4 marks)

End of Paper

APPENDIX

A. Differentiation Rules

$\frac{d}{dx}[x^n] = nx^{n-1}$; n is any real number

$\frac{d}{dx}[f(x).g(x)] = f(x)g'(x) + f'(x)g(x)$; The Product Rule

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$; The Quotient Rule

$\frac{d}{dx}[f(g(x))] = f'(g(x)).g'(x)$; The Chain Rule

$\frac{d}{dx}[g(x)]^n = n[g(x)]^{n-1}.g'(x)$; The power rule combined with the chain rule:

$\frac{d}{dx}[\sin x] = \cos x$ $\quad$ $\frac{d}{dx}[\cos x] = -\sin x$ $\quad$ $\frac{d}{dx}[\tan x] = \sec^2 x$

$\frac{d}{dx}[\sec x] = \sec x \tan x$ $\quad$ $\frac{d}{dx}[\cot x] = -\csc^2 x$ $\quad$ $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\frac{d}{dx}[e^x] = e^x$ $\quad$ $\frac{d}{dx}[\ln x] = \frac{1}{x}; \quad x > 0$

B. Basic Integration Formulas

$\int cf(x)\,dx = c\int f(x)\,dx$ $\quad$ $\int k.dx = kx + C$

$\int [f(x) \pm g(x)]dx = \int f(x)dx \pm \int g(x)dx$ $\quad$ $\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$

$\int e^x\,dx = e^x + C$ $\quad$ $\int \frac{1}{x}dx = \ln|x| + C$

Integration by-parts: $\int u\,dv = uv - \int v\,du$

Volume (disk) $= \pi\int_a^b (f(x))^2\,dx$ $\quad$ **Area** $= \int_a^b (f(x) - g(x))dx$

Volume (washer) $= \pi\int_a^b \left[(f(x))^2 - (g(x))^2\right]dx$